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NBV-003-1261002 Seat No. ________

M. Phil. (Sem. I) (CBCS) Examination

April/May – 2017

Mathematics : CMT-10002

(Combinatorics & Graph Theory)

(New Course)

Faculty Code : 003
Subject Code : 1261002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt all the questions.

(2) Each question carries equal marks.

1 Attempt the followings : (any seven)

(1) Explain with example the principle of Exclusion and Inclusion.

(2) State only the pigeonhole principle.

(3) Define : Eulerian graph. Does every Eulerian graph is bipartite ? Justify your answer.

(4) Enumerate the number of edges in K_8.

(5) Enumerate the number of edges in $K_{3,\ 10}$.

(6) Define : Trail and Path.

(7) Write the adjacency matrix for the Petersen graph.

(8) Define : Incidence matrix of a graph and prepare the same for the graph C_5.

(9) Draw a graph which is Complete, regular, Eulerian and Hamiltonian and whose domination number is 1.

(10) Define : Forest and Tree.

2 Attempt the followings : (any two)

(a) Define : Order and Size of a graph. Also prove that the sum of degrees of vertices of a graph is twice the size of that graph.

(b) Define : Bipartite graph. Also prove that a non empty graph with at least two vertices is bipartite iff it has no odd cycles.

(c) Prove that : For a graph G with n-vertices the following are equivalent :

(i) G is a tree.

(ii) G is an acyclic graph with $n-1$ edges.

(iii) G is a connected graph with $n-1$ edges.

3 Attempt the followings :

(a) Prove that any simple graph G having all the vertices with degree at least two then G contains a cycle.

(b) Prove that there is precisely one path between any two vertices of a tree.

OR

3 (a) Define : Hamiltonian path and Hamiltonian graph. Also prove that if G is a simple graph with n-vertices (where $n \geq 3$) and degree of every vertex is at least $n/2$ then G is Hamiltonian.

(b) Find the number of integers between 1 and 5000 inclusive that are not divisible by 5, 8 or 10.

4 Attempt the followings :

(a) Define the terms : Proper coloring and chromatic number. Also prove that if G is a non empty graph then $X(G) = 2$ iff G is bipartite.

(b) Define the terms : Plane graph and Planar graph. Also Draw Kuratowski's two graphs.

5 Attempt the followings : (any two)

(a) Define :

(i) Directed graph

(ii) In-degree

(iii) Out-degree

(iv) Spanning subgraph

(v) Edge chromatic number.

(b) Define : Minimal dominating set. Also prove that a dominating set S is minimal iff for each vertex $u \in S$ one of the following two conditions holds :

(i) u is an Isolate of S.

(ii) There exists a vertex $\vartheta \in V - S$ for which $N(\vartheta) \cap S = \{u\}$.

(c) Prove that : Every connected graph $|V| \geq 2$ has a dominating set S whose complement $V–S$ is also a dominating set.

(d) Prove that : If G is a graph with no isolated vertices, then the complement $V–S$ of every minimal dominating set S is a dominating set.